Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **9FM0/4A**

Further Mathematics

Advanced

PAPER 4A: Further Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over

Pearson

DO NOT WRITE IN THIS AREA

1. **In this question you must show detailed reasoning.**

Without performing any division, explain why $n = 20\,210\,520$ is divisible by 66

(4)

11 and 3 are prime divisors of 66 ① ← so we can use divisibility tests to check if n is divisible by 66

$66 = 11 \times 6 = \underline{11} \times \underline{3} \times 2$

$n = 20210520$

$2 - 0 + 2 - 1 + 0 - 5 + 2 - 0 = 0 \times 11$ hence n is divisible by 11 ①

$2 + 0 + 2 + 1 + 0 + 5 + 2 + 0 = 12 = 4 \times 3$ hence n is divisible by 3 ①

n is even and so is divisible by 2.

$\therefore$ n is divisible by 2, 3 and 11 and so is also divisible by $2 \times 3 \times 11 = 66$. ①

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

(Total for Question 1 is 4 marks)

2. A binary operation ★ on the set of non-negative integers, $\mathbb{Z}_0^+$, is defined by

$$m \star n = |m - n| \qquad m, n \in \mathbb{Z}_0^+$$

(a) Explain why $\mathbb{Z}_0^+$ is closed under the operation ★ **(1)**

(b) Show that 0 is an identity for $(\mathbb{Z}_0^+, \star)$ **(2)**

(c) Show that all elements of $\mathbb{Z}_0^+$ have an inverse under ★ **(2)**

(d) Determine if $\mathbb{Z}_0^+$ forms a group under ★, giving clear justification for your answer. **(3)**

(a) For $\underbrace{m, n \in \mathbb{Z}_0^+}_{\text{pair of non-negative integers}}$, $\underbrace{m-n \in \mathbb{Z}}_{\text{difference is also an integer}}$ and so $\underbrace{|m-n| \in \mathbb{Z}_0^+}_{\text{so magnitude / modulus will be a non-negative integer}}$ is closed under $*$

(b) For $m \in \mathbb{Z}_0^+$, $0 * m = |0-m| = |-m| = m$ (1)

$m * 0 = |m-0| = |m| = m$ ← need to check both directions.

Hence, 0 is the identity element. (1)

(c) For $m \in \mathbb{Z}_0^+$, we want $\underbrace{|m-n| = 0}_{\text{so that } m * n = e \text{ (identity)}} \implies n = m$ (1)

As $n = m$, and hence $|m-m| = 0$ for all $m \in \mathbb{Z}_0^+$, each m is self-inverse. (1)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 2 continued

(d) Groups have an identity, inverse, closure and associativity.

Associative if $m * (n * p) = (m * n) * p$

$1 * (2 * 3) = 1 * |2-3| = 1 * 1 = 0$ (1)

$(1 * 2) * 3 = |1-2| * 3 = |1-3| = 2$ (1)

$1 * (2 * 3) \neq (1 * 2) * 3$ hence not associative, so does not form a group. (1)

(Total for Question 2 is 8 marks)

3. (a) Use the Euclidean Algorithm to find integers a and b such that

$$125a + 87b = 1$$

(5)

(b) Hence write down a multiplicative inverse of 87 modulo 125

(1)

(c) Solve the linear congruence

$$87x \equiv 16 \pmod{125}$$

(2)

(a) $125 = 87 \times 1 + 38$

$87 = 38 \times 2 + 11$ (1)

$38 = 11 \times 3 + 5$ (1)

$11 = 5 \times 2 + 1$ (1) ← stop when remainder = 1

Apply steps in reverse:

$1 = 11 - 5 \times 2$

$1 = 11 - (38 - 11 \times 3) \times 2$

$1 = 11 - (-11 \times 3) \times 2 - 38 \times 2$

$1 = 7 \times 11 - 38 \times 2$

$1 = (87 - 38 \times 2) \times 7 - 38 \times 2$

$1 = 87 \times 7 - 36 \times 16$ (1)

$1 = 87 \times 7 - (125 - 87 \times 1) \times 16$

$1 = -16 \times 125 + 23 \times 87$ (1)

$\therefore a = -16, \quad b = 23$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 3 continued

(b) From (a) $23 \times 87 = 1 \pmod{125}$ so multiplicative inverse of 87 is 23. 1

(c) $x \equiv 23 \times 16 \pmod{125}$ 1

$x \equiv 368 \equiv 118 \pmod{125}$ 1

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

4. Let G be a group of order $46^{46} + 47^{47}$

Using Fermat's Little Theorem and explaining your reasoning, determine which of the following are possible orders for a subgroup of G

(i) 11

(ii) 21

(7)

Order of a subgroup must divide the order of a group by Lagrange's Theorem.

(i) So check if 11 divides $46^{46} + 47^{47}$

By FLT : $a^{11-1} = a^{10} \equiv 1 \pmod{11}$

$46^{46} + 47^{47} \equiv 2^{4\times10+6} + 3^{4\times10\times7} \pmod{11}$ ①

$\equiv 2^6 + 3^7 \pmod{11}$

$\equiv 64 + (3^3)^2 \times 3 \pmod{11}$

$\equiv 9 + 5^2 \times 3 \pmod{11}$

$\equiv 84 \pmod{11}$

$\equiv 7 \pmod{11}$ ①

Hence 11 is not a divisor of $46^{46} + 47^{47}$ so not a possible order for a subgroup ①

(ii) $21 = 7 \times 3$ so check for factors of 7 and 3

$a^{3-1} \equiv a^2 \equiv 1 \pmod 3$

$a^{7-1} \equiv a^6 \equiv 1 \pmod 3$ ①

DO NOT WRITE IN THIS AREA

Question 4 continued

$$46^{46} + 47^{47} \equiv 1^{46} + 2^{47} \pmod 3$$
$$\equiv 1 + 2^{2\times 23+1} \pmod 3$$
$$\equiv 1 + 2^{1} \pmod 3$$
$$\equiv 3 \pmod 3$$
$$\equiv 0 \pmod 3 \quad (1)$$

$$46^{46} + 47^{47} \equiv 4^{46} + (-2)^{47} \pmod 3$$
$$\equiv 4^{6\times 7+4} + (-2)^{6\times 7+5} \pmod 3$$
$$\equiv 4^{4} + (-2)^{5} \pmod 3$$
$$\equiv 16^{2} - 32 \pmod 3$$
$$\equiv 9^{2} - 4 \pmod 3$$
$$\equiv 81 - 4 \pmod 3$$
$$\equiv 77 \pmod 3$$
$$\equiv 0 \pmod 3 \quad (1)$$

As $46^{46} + 47^{47}$ is divisible by 3 and 7 it is divisible by 21, hence this is a possible order for a subgroup (1)

(Total for Question 4 is 7 marks)

5. The point P in the complex plane represents a complex number z such that

$$|z+9| = 4|z-12\mathrm{i}|$$

Given that, as z varies, the locus of P is a circle,

(a) determine the centre and radius of this circle.

(6)

(b) Shade on an Argand diagram the region defined by the set

$$\{z \in \mathbb{C} : |z+9| < 4|z-12\mathrm{i}|\} \cap \left\{z \in \mathbb{C} : -\frac{\pi}{4} < \arg\left(z - \frac{3+44\mathrm{i}}{5}\right) < \frac{\pi}{4}\right\}$$

(4)

(a) $z = x + yi$ substitute, group real and imaginary terms.

$|x + 9 + iy| = 4|x + (y-12)i|$ ①

$(x+9)^2 + (y)^2$ ① $= 16[x^2 + (y-12)^2]$ ① ← square terms, considering real / im separately

$x^2 + 18x + 81 + y^2 = 16x^2 + 16y^2 - 384y + 2304$

$15x^2 - 18x + 15y^2 - 384y = -2223$

$5x^2 - 6x + 5y^2 - 128 = -741$

$\left(x - \frac{3}{5}\right)^2 - \left(\frac{3}{5}\right)^2 + \left(y - \frac{64}{5}\right)^2 - \left(\frac{64}{5}\right)^2 = -\frac{741}{5}$

$\left(x - \frac{3}{5}\right)^2 + \left(y - \frac{64}{5}\right)^2 = 16$ ①

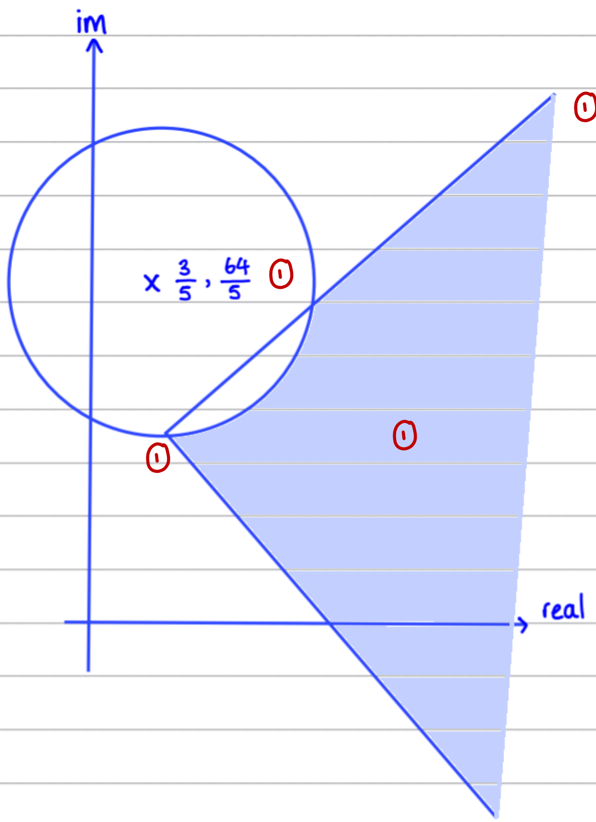
$\therefore$ Centre $= \frac{3}{5} + \frac{64}{5}i$ ① and radius $= \sqrt{16} = 4$ ①

DO NOT WRITE IN THIS AREA

Question 5 continued

(b) Use centre and radius from (a) to sketch circle.

$z \in \mathbb{C} : -\frac{\pi}{4} < \arg\left(z - \frac{3+44i}{5}\right) < \frac{\pi}{4}$ is a range between $-\frac{\pi}{4}$ and $\frac{\pi}{4}$ (45° either side of the horizontal axis) from the bottom of the circle



Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6. A recurrence system is defined by

$$u_{n+2} = 9(n+1)^2 u_n - 3u_{n+1} \qquad n \geqslant 1$$

$$u_1 = -3,\ u_2 = 18$$

Prove by induction that, for $n \in \mathbb{N}$,

$$u_n = (-3)^n n!$$

(6)

Consider the case when $n = 1$

$n = 1 \implies u_1 = (-3)^1 \times 1! = -3$

$n = 2 \implies u_2 = (-3)^2 \times 2! = 18$

$\therefore$ True for $n=1$ and $n=2$. 1

Assume true for $n=k$ and $n=k+1$, such that:

$u_k = (-3)^k k!$ and $u_{k+1} = (-3)^{k+1}(k+1)!$ 1

substitute into the recurrence system

Now consider $n = k+2$

$u_{k+2} = 9(k+1)^2\left[(-3)^k k!\right] - 3\left[(-3)^{k+1}(k+1)!\right]$ 1

$u_{k+2} = (-3)^k k!\left[9(k+1)^2 - 3(-3)(k+1)\right]$ factorise 1

$u_{k+2} = (-3)^k k!\left[9(k+1)^2 + 9(k+1)\right]$

$u_{k+2} = (-3)^k k!\left[9(k+1)(k+1+1)\right]$

$u_{k+2} = (-3)^k \times (-3)^2 \times (k+1)(k+2)k!$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 6 continued

$$U_{k+2} = (-3)^{k+2}(k+2)! \quad (1)$$

$\therefore$ True for $n = k+2$.

Hence if true for $n=k$, $n=k+1$ then true for $n=k+2$.

As also true for $n=1$ and $n=2$, then true for all

$n \in \mathbb{N}$ by mathematical induction (1)

Question 6 continued

Question 6 continued

(Total for Question 6 is 6 marks)

7. **In this question you must show all stages of your working.**

You must not use the integration facility on your calculator.

$$I_n = \int t^n \sqrt{4 + 5t^2}\, dt \qquad n \geqslant 0$$

(a) Show that, for $n > 1$

$$I_n = \frac{t^{n-1}}{5(n+2)}\left(4 + 5t^2\right)^{\frac{3}{2}} - \frac{4(n-1)}{5(n+2)} I_{n-2}$$

(5)

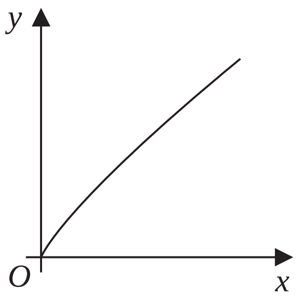


Figure 1

The curve shown in Figure 1 is defined by the parametric equations

$$x = \frac{1}{\sqrt{5}} t^5 \qquad y = \frac{1}{2} t^4 \qquad 0 \leqslant t \leqslant 1$$

This curve is rotated through 2π radians about the x-axis to form a hollow open shell.

(b) Show that the external surface area of the shell is given by

(5)

$$\pi \int_0^1 t^7 \sqrt{4 + 5t^2}\, dt$$

Using the results in parts (a) and (b) and making each step of your working clear,

(c) determine the value of the external surface area of the shell, giving your answer to 3 significant figures.

(5)

(a) $I_n = \int t^n \sqrt{4 + 5t^2}\, dt$

$I_n = \int t^{n-1} \times t \sqrt{4 + 5t^2}\, dt$

separate into t^{n-1}, t^1 terms
$(x^a \times x^b = x^{a+b})$

DO NOT WRITE IN THIS AREA

Question 7 continued

Use integration by parts.

$u = t^{n-1}$ $\qquad v' = (4+5t^2)^{\frac{1}{2}}$

$u' = (n-1)t^{n-2}$ $\qquad v = \frac{2}{3} \times \frac{1}{10}(4+5t^2)^{\frac{3}{2}}$

$\int uv' = uv - \int vu'$:

$$I_n = t^{n-1} \times \frac{1}{15}(4+5t^2)^{\frac{3}{2}} - \int (n-1)t^{n-2} \times \frac{1}{15}(4+5t)^{\frac{3}{2}}\,dt \quad ①$$

①

$$I_n = t^{n-1} \times \frac{1}{15}(4+5t^2)^{\frac{3}{2}} - \frac{1}{15}(n-1)\int t^{n-2}(4+5t^2)^{\frac{1}{2}} \times (4+5t^2)\,dt$$

$$I_n = \frac{t^{n-1}}{15}(4+5t^2)^{\frac{3}{2}} - \frac{4(n-1)}{15}\int t^{n-2}(4+5t^2)^{\frac{1}{2}}\,dt \quad ①$$

$$- \frac{5(n-1)}{15}\int t^n(4+5t)^{\frac{1}{2}}\,dt$$

$$15I_n = t^{n-1}(4+5t^2)^{\frac{3}{2}} - 4(n-1)I_{n-2} - 5(n-1)I_n \quad ①$$

$$15I_n + 5(n-1)I_n = t^{n-1}(4+5t^2)^{\frac{3}{2}} - 4(n-1)I_{n-2}$$

$(15 + 5n - 5)I_n = (5n+10)I_n$

$$5(n+2)I_n = t^{n-1}(4+5t^2)^{\frac{3}{2}} - 4(n-1)I_{n-2}$$

$$I_n = \frac{t^{n-1}}{5(n+2)}(4+5t^2)^{\frac{3}{2}} - \frac{4(n-1)}{5(n+2)}I_{n-2} \quad ①$$

Question 7 continued

(b) Surface Area $= 2\pi \int_0^1 y \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$ ①

$x = \frac{1}{\sqrt{5}} t^5 \Rightarrow \frac{dx}{dt} = 5 \times \frac{1}{\sqrt{5}} t^4 = \frac{5}{\sqrt{5}} t^4$

$y = \frac{1}{2} t^4 \Rightarrow \frac{dy}{dt} = 4 \times \frac{1}{2} t^3 = 2t^3$ ①

$SA = 2\pi \int_0^1 \frac{1}{2} t^4 \sqrt{\left(\frac{5}{\sqrt{5}} t^4\right)^2 + (2t^3)^2}\, dt$ ①

$\left(\frac{5}{\sqrt{5}}\right)^2 = \frac{25}{5} = 5$

$SA = 2\pi \int_0^1 \frac{1}{2} t^4 \sqrt{5t^8 + 4t^6}\, dt$

$SA = \frac{1}{2} \times 2\pi \int_0^1 t^4 \times t^{6 \times \frac{1}{2}} \sqrt{5t^2 + 4}\, dt$ ①

$SA = \pi \int_0^1 t^7 \sqrt{5t^2 + 4}\, dt$ ①

(c) $[I_1]_0^1 = \left[\frac{1}{15}(4 + 5t^2)^{\frac{3}{2}}\right]_0^1$

$[I_1]_0^1 = \frac{27}{15} - \frac{8}{15} = \frac{19}{15}$ ①

$\int t^7 \sqrt{4 + 5t^2}\, dt = \left[\frac{t^6}{5 \times 9}(4 + 5t^2)^{\frac{3}{2}}\right]_0^1 - \frac{4 \times 6}{5 \times 9} [I_5]_0^1$ ①

sub in I_3

$= \frac{3}{5} - \frac{8}{15}\left(\left[\frac{t^4}{5 \times 7}(4 + 5t^2)^{\frac{3}{2}}\right]_0^1 - \frac{4 \times 4}{5 \times 7}[I_3]_0^1\right)$

sub in I_0

$= \frac{3}{5} - \frac{8}{15}\left(\frac{27}{35} - \frac{16}{35}\left(\left[\frac{t^2}{5 \times 5}(4 + 5t^2)^{\frac{3}{2}}\right]_0^1 - \frac{4 \times 2}{5 \times 5}[I_0]_0^1\right)\right)$ ①

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

$\therefore$ surface area $= \pi\left[\frac{3}{5} - \frac{8}{15}\left(\frac{27}{35} - \frac{16}{35}\left(\frac{27}{25} - \frac{8}{25} \times \frac{19}{15}\right)\right)\right]_0^1$ 1

$= 1.11$ (3.s.f) 1

(Total for Question 7 is 15 marks)

8.

$$\mathbf{A} = \begin{pmatrix} 5 & -2 & 5 \\ 0 & 3 & p \\ -6 & 6 & -4 \end{pmatrix} \quad \text{where } p \text{ is a constant}$$

Given that $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ is an eigenvector for **A**

(a) (i) determine the eigenvalue corresponding to this eigenvector **(1)**

(ii) hence show that $p = 2$ **(2)**

(iii) determine the remaining eigenvalues and corresponding eigenvectors of **A** **(7)**

(b) Write down a matrix **P** and a diagonal matrix **D** such that $\mathbf{A} = \mathbf{PDP}^{-1}$ **(1)**

(c) (i) Solve the differential equation $\dot{u} = ku$, where k is a constant. **(2)**

With respect to a fixed origin O, the velocity of a particle moving through space is modelled by

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

By considering $\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \mathbf{P}^{-1} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ so that $\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \mathbf{P}^{-1} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}$

(ii) determine a general solution for the displacement of the particle. **(4)**

(a)(i) $\begin{bmatrix} 5 & -2 & 5 \\ 0 & 3 & p \\ -6 & 6 & -4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 5\times 2 + -2\times 1 + 5\times -2 \\ 0\times 2 + 3\times 1 + p\times -2 \\ -6\times 2 + 6\times 1 + -4\times -2 \end{bmatrix}$

$= \begin{bmatrix} -2 \\ 3-2p \\ 2 \end{bmatrix}$

Question 8 continued

Eigenvalue λ satisfies $Av = \lambda v$

$$\begin{bmatrix} -2 \\ 3-2p \\ 2 \end{bmatrix} = \lambda \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \quad \therefore \lambda = -1 \quad (1)$$

(Av) (λv)

(a)(ii) $3 - 2p = -1$

$4 = 2p$

$2 = p$ (1)

(a)(iii) $\det \begin{bmatrix} 5-\lambda & -2 & 5 \\ 0 & 3-\lambda & p \\ -6 & 6 & -4-\lambda \end{bmatrix} = 0$ $p = 2$

$(5-\lambda)[(3-\lambda)(-4-\lambda) - 6p] - -2[- -6p] + 5[6(3-\lambda)] = 0$

$(5-\lambda)[-12 + \lambda + \lambda^2 - 6p] + 12p + 90 - 5\lambda = 0$ (1)

$-60 + 5\lambda + 5\lambda^2 - 30p + 12\lambda - \lambda^2 - \lambda^3 + 12\lambda + 24 + 90 - 5\lambda = 0$

$-\lambda^3 + 4\lambda^2 - \lambda - 6 = 0$

$\lambda^3 - 4\lambda^2 + \lambda + 6 = 0$ (1)

$(\lambda+1)(\lambda^2 - 5\lambda + 6) = 0$

$(\lambda+1)(\lambda-2)(\lambda-3) = 0$ so Eigenvalues are $-1, 2, 3$ (1)

Question 8 continued

When $\lambda = 2$: $\begin{bmatrix} 5 & -2 & 5 \\ 0 & 3 & 2 \\ -6 & 6 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$5x - 2y + 5z = 2x \Rightarrow 3x - 2y + 5z = 0$ 1

$3y + 2z = 2y \Rightarrow y + 2z = 0$ 2

$-6x + 6y - 4z = 2z \Rightarrow -6x + 6y - 6z = 0$ 1 3

$y = -2z \Rightarrow 3x + 4z + 5z = 0$ use 1 to eliminate y

$3x = -9z \Rightarrow x = -3z$

$\therefore$ when $z = 1$, $x = -3$ and $y = -2$

When $\lambda = 3$: $\begin{bmatrix} 5 & -2 & 5 \\ 0 & 3 & 2 \\ -6 & 6 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 3\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$5x - 2y + 5z = 3x \Rightarrow 2x - 2y + 5z = 0$ 1

$3y + 2z = 3y \Rightarrow 2z = 0$ 2

$-6x + 6y - 4z = 3z \Rightarrow -6x + 6y - z = 0$ 3 1

$z = 0 \Rightarrow 2x - 2y = 0 \Rightarrow x = y$

$\therefore$ when $x = 1$, $y = 1$ and $z = 0$

Eigenvectors are $\begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$ 1 and $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ 1

DO NOT WRITE IN THIS AREA

Question 8 continued

(b) e.g $P = \begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ -2 & 1 & 0 \end{pmatrix}$ $D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

each column is an eigenvector. leading diagonal is eigenvalues.

(c)(i) $\dot{u} = ku$

$$\int \frac{1}{u}\,du = k\int dt$$

$\ln u = kt + c$ (1)

$\therefore u = Ae^{kt}$ (1)

(c)(ii) $\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = PDP^{-1}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ (1)

$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = P^{-1}\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = D\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -u \\ 2v \\ 3w \end{pmatrix}$ (1)

$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} Ae^{-t} \\ Be^{2t} \\ Ce^{3t} \end{pmatrix}$ ← using part (i) (1)

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = P\begin{pmatrix} Ae^{-t} \\ Be^{2t} \\ Ce^{3t} \end{pmatrix} = \begin{pmatrix} 2Ae^{-t} + 3Be^{2t} + Ce^{3t} \\ Ae^{-t} + 2Be^{2t} + Ce^{3t} \\ -2Ae^{-t} - Be^{2t} \end{pmatrix}$ (1)

Question 8 continued

(Total for Question 8 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS